Reduced Hamiltonian for intersecting shells and Hawking radiation

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Abstract. We consider the dynamics of one or more self gravitating shells of matter in a centrally symmetric gravitational field in the Painlevé family of gauges. We give the reduced hamiltonian for two intersecting shells, both massless and massive. Such a formulation is applied to the computation of the semiclassical action of two intersecting shells. The relation of the imaginary part of the space-part of the action to the computation of the Bogoliubov coefficients is revisited.

1. Introduction

In this paper we shall study the dynamics of one or more self-gravitating spherical shells of matter subject to a centrally symmetric gravitational field and the application of the ensuing formalism to the semiclassical treatment of Hawking radiation. Such field of research was started by the papers by Kraus and Wilczek [1, 2]. The main results we shall report here are the extension of the treatment to one massive shell of matter and also the extension to two or more massive shells of matter which in the time development can also intersect [3]. The main appeal of the approach of [1, 2] is that energy conservation is taken exactly into account and thus the back reaction effects can be computed.

We shall keep the formalism for more that one shell as close as possible to the original formalism of [1] which can be summarized in words that we shall adopt a Painlevé gauge.

The extraction of the reduced hamiltonian is often classified as a very complicated procedure. We shall show that by introducing a "generating function" the derivation can be drastically simplified and also by the same token it can be extended to deal not only with massive shells but also with a finite number of massive shells. The treatment of massless shells is just a particular case.

In Sec.2 we shall give the derivation of the reduced action in the case of one shell while proving the independence of the canonical momentum within the Painlevé class of gauges. In Sec.3 we shall discuss the equations of motion. In Sec.4 we work out the analytic properties of the conjugate momentum p_c which appears in the reduced action and in Sec.5 we shall briefly describe the extension of the treatment to more than one shell. An application of the results is given in Sec.6 giving a derivation within such a formalism of the Dray-'t Hooft- Redmount exchange relations. An important integrability relation is reported in Sec.7 which allows to compute the imaginary part of the space integral of the canonical momenta for two shells. In Sec.8 we revisit the role of the imaginary part of the canonical momentum in determining the

Bogoliubov coefficients and thus the most important features of Hawking radiation. In Sec.9 we give some concluding remarks.

2. The reduced action

As usual we write the metric for a spherically symmetric configuration in the ADM form

$$ds^2 = -N^2 dt^2 + L^2(dr + N^r dt)^2 + R^2 d\Omega^2. \tag{1}$$

were following [1, 2, 4, 5] we shall choose the functions N, N^r, L, R as continuous functions of the coordinates. We shall work on a finite region of space time $(t_i, t_f) \times (r_0, r_m)$. On the two initial and final surfaces we give the intrinsic metric by specifying $R(r, t_i)$ and $L(r, t_i)$ and similarly $R(r, t_f)$ and $L(r, t_f)$.

The complete action in hamiltonian form, boundary terms included is [1, 2, 6, 7]

$$S = S_{shell} + \int_{t_i}^{t_f} dt \int_{r_0}^{r_m} dr(\pi_L \dot{L} + \pi_R \dot{R} - N\mathcal{H}_t - N^r \mathcal{H}_r) + \int_{t_i}^{t_f} dt \, (-N^r \pi_L L + \frac{NRR'}{L})\Big|_{r_0}^{r_m} \tag{2}$$

where

$$S_{shell} = \int_{t_i}^{t_f} dt \; \hat{p} \; \dot{\hat{r}}. \tag{3}$$

$\hat{r}$ is the shell position and $\hat{p}$ its canonical conjugate momentum. $\mathcal{H}_r$ and $\mathcal{H}_t$ are the constraints

$$\mathcal{H}_r = \pi_R R' - \pi_L' L - \hat{p} \; \delta(r - \hat{r}), \tag{4}$$

$$\mathcal{H}_t = \frac{RR''}{L} + \frac{R'^2}{2L} + \frac{L\pi_L^2}{2R^2} - \frac{RR'L'}{L^2} - \frac{\pi_L \pi_R}{R} - \frac{L}{2} + \sqrt{\hat{p}^2 L^{-2} + m^2} \; \delta(r - \hat{r}). \tag{5}$$

Action (2) is immediately generalized to a finite number of shells. The shell action as given by eqs.(3,4,5) refers to a dust shell even though generalizations to more complicated equations of state have been considered [7, 8, 9]. The boundary terms are those given in the paper by Hawking and Hunter [6] and will play a very important role in the following. The function F [3]

$$F = RL\sqrt{\left(\frac{R'}{L}\right)^2 - 1 + \frac{2\mathcal{M}}{R}} + RR' \log\left(\frac{R'}{L} - \sqrt{\left(\frac{R'}{L}\right)^2 - 1 + \frac{2\mathcal{M}}{R}}\right) \tag{6}$$

has the remarkable property of generating the conjugate momenta as solutions of the constraints as follows

$$\pi_L = \frac{\delta F}{\delta L} = \frac{\partial F}{\partial L} \tag{7}$$

$$\pi_R = \frac{\delta F}{\delta R} = \frac{\partial F}{\partial R} - \frac{\partial}{\partial r}\frac{\partial F}{\partial R'} \tag{8}$$

where $\mathcal{M}$ is a mass which is constant in r except at the shell position $\hat{r}$ and that we shall denote by H for r above all the shell positions and by M for r below all the shell positions. We shall adopt a Painlevé gauge defined by $L = 1$ everywhere. With regard to the remaining freedom in the choice of the gauge we shall choose $R = r$ except for a deformation region near the shell positions. Such deformation is unavoidable because the constraints impose a discontinuity in the derivative $R'(r)$ at $r = \hat{r}$ as we shall see in the following. In the Painlevé gauges F becomes

$$F = RW(R, R', \mathcal{M}) + RR'(\mathcal{L}(R, R', \mathcal{M}) - \mathcal{B}(R, \mathcal{M})) \tag{9}$$

where

$$W(R, R', \mathcal{M}) = \sqrt{R'^2 - 1 + \frac{2\mathcal{M}}{R}}; \quad \mathcal{L}(R, R', \mathcal{M}) = \log(R' - W(R, R', \mathcal{M})) \tag{10}$$

and

$$\mathcal{B}(R, \mathcal{M}) = \sqrt{\frac{2\mathcal{M}}{R}} + \log\left(1 - \sqrt{\frac{2\mathcal{M}}{R}}\right) \tag{11}$$

where we exploited the freedom of adding to (6) a total derivative, thus gaining for F the useful property of vanishing wherever $R' = 1$. For $L = 1$ eqs.(7,8) become

$$\pi_L = R\sqrt{R'^2 - 1 + \frac{2\mathcal{M}}{R}} \equiv RW; \qquad \pi_R = \frac{[RR'' + R'^2 - 1 + \mathcal{M}/R]}{W}. \tag{12}$$

With regard to $R(r, t)$ one can choose several gauges, within the Painlevé family.

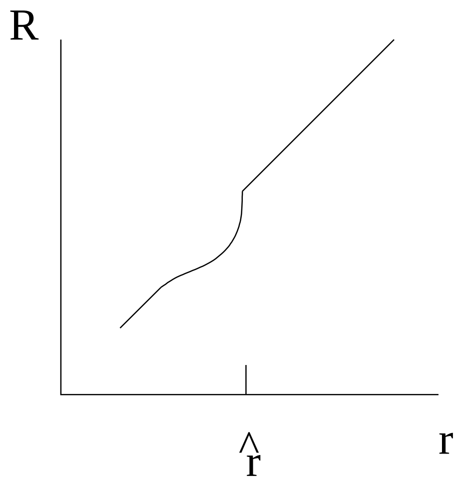

Figure 1. Outer gauge

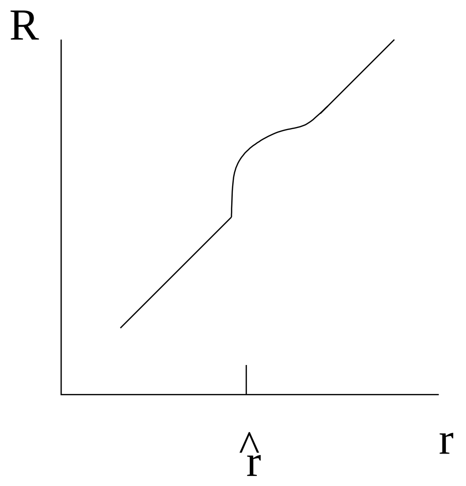

Figure 2. Inner gauge

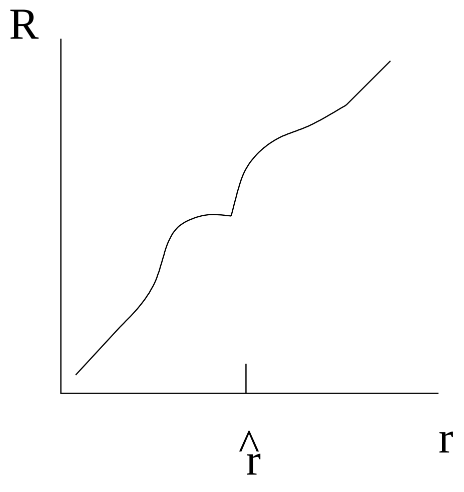

Figure 3. Generic gauge

Typical are the "outer gauge" characterized by $R(r, t) = r$ for $r \geq \hat{r}(t)$ and shown in Fig.1 and the inner gauge characterized by $R(r, t) = r$ for $r \leq \hat{r}(t)$ shown in Fig.2, but there are also

more general choices as shown in Fig.3. We stress that such deformation g is not related to the thickness of the shell which is always zero.

Using the solutions (7,8) of the constraints one obtains in the outer gauge the following reduced [3] action where only the shell position $\hat{r}$ appears as degree of freedom

$$\int_{t_i}^{t_f}\left(p_c\,\dot{\hat{r}}-\dot{M}(t)\int_{r_0}^{\hat{r}(t)}\frac{\partial F}{\partial M}dr+(-N^r\pi_L+NRR')|_{r_0}^{r_m}\right)dt. \tag{13}$$

The p_c is easily computed

$$p_c=\hat{r}(\Delta\mathcal{L}-\Delta\mathcal{B}) \tag{14}$$

where $\Delta\mathcal{L}=\mathcal{L}(\hat{r}+\varepsilon)-\mathcal{L}(\hat{r}-\varepsilon)$ and similarly for $\Delta\mathcal{B}$. In deriving eq.(14) we used the consequences of the constraints (4,5)

$$\Delta R'=-\frac{V}{R};\quad\text{where}\;\; V=\sqrt{\hat{p}^2+m^2};\qquad \Delta\pi_L=-\hat{p}. \tag{15}$$

In the outer gauge we find

$$p_c=\sqrt{2M\,\hat{r}}-\sqrt{2H\,\hat{r}}-\hat{r}\log\left(\frac{\hat{r}+\sqrt{\hat{p}^2+m^2}-\hat{p}-\sqrt{2H\,\hat{r}}}{\hat{r}-\sqrt{2M\,\hat{r}}}\right) \tag{16}$$

with $\hat{p}$ given implicitly by

$$H-M=V+\frac{m^2}{2\hat{r}}-\hat{p}\sqrt{\frac{2H}{\hat{r}}};\qquad V=\sqrt{\hat{p}^2+m^2}. \tag{17}$$

This is the result obtained by Friedman, Louko, Winters-Hilt [4] through a limit procedure in which the support of the deformation function goes to zero but actually it is independent of the deformation. For working out the dynamics of one shell the limit procedure is all right, but in dealing with two or more shell it is important to have a deformation on a finite range because otherwise a limit procedure would give a result dependent on the order in which the two limits, deformation and $\hat{r}_1\rightarrow\hat{r}_2$, are taken.

Similarly one can compute using the general formula the reduced canonical momentum of the system in the inner gauge

$$p_c^i=\sqrt{2M\,\hat{r}}-\sqrt{2H\,\hat{r}}-\hat{r}\log\left(\frac{\hat{r}-\sqrt{2H\,\hat{r}}}{\hat{r}-V+\hat{p}-\sqrt{2M\hat{r}}}\right) \tag{18}$$

and $\hat{p}$, again determined by the discontinuity equation (15), is given now by the implicit equation

$$H-M=V-\frac{m^2}{2\hat{r}}-\hat{p}\sqrt{\frac{2M}{\hat{r}}};\qquad V=\sqrt{\hat{p}^2+m^2}. \tag{19}$$

The two reduced canonical momenta p_c, p_c^i appear to be completely different but they can be proven to be the same function of $\hat{r}$. One can consider also the more general gauges depicted in Fig.3 and it can be proven [10] that p_c is always the same. However in inner gauge a term $\dot{H}(t)$ appears in the reduced action while in the more general gauges both a term $\dot{M}$ and a term $\dot{H}$ appear in the action.

The boundary term given in eq.(2) is equivalent to

$$-\,HN(r_m)+MN(r_0). \tag{20}$$

3. Equations of motion

In deriving the equation of motion from action (13) one can consider $M(t) = M$, the interior mass, as a datum of the problem and vary H to obtain

$$\dot{\hat{r}}\frac{\partial p_c}{\partial H} - N(r_m) = 0 \tag{21}$$

which using the expression of p_c and the relation between N and N^r imposed by the gravitational equations [3, 4] can be written as

$$\dot{\hat{r}} = \frac{\hat{p}}{V}N(\hat{r}) - N^r(\hat{r}) = \left(\frac{\hat{p}}{V} - \sqrt{\frac{2H}{\hat{r}}}\right)N(\hat{r}). \tag{22}$$

Alternatively one can consider H, the total energy as a datum of the problem and vary $M(t)$. The calculation is far more complicated due to the presence of $\dot{M}$ in the action (13), but using the equations for the gravitational field one reaches [3] the same equation of motion (22). In addition a consequence of the gravitational equations of motion is the constancy in time of $M(t)$ and $H(t)$.

4. Analytic properties of p_c

We saw that in the outer gauge p_c is given by (16,17). The solution of eq.(17) for $\hat{p}$ is

$$\frac{\hat{p}}{\hat{r}} = \frac{A\sqrt{\frac{2H}{\hat{r}}} \pm \sqrt{A^2 - (1-\frac{2H}{\hat{r}})\frac{m^2}{\hat{r}^2}}}{1-\frac{2H}{\hat{r}}} \tag{23}$$

where

$$A = \frac{H-M}{\hat{r}} - \frac{m^2}{2\hat{r}^2}. \tag{24}$$

If we want $\hat{p}$ to describe an outgoing shell we must choose the plus sign in front of the square root. Moreover the shell reaches $r = +\infty$ if and only if $H - M > m$ as expected.

The logarithm in p_c, eq.(16), has branch points at zero and infinity and thus we must investigate for which values or $\hat{r}$ such values are reached. At $\hat{r} = 2H$, $\hat{p}$ has a simple pole with positive residue. Thus the numerator in the argument of the logarithm in p_c goes to zero and below $2H$ it becomes

$$\hat{r} - V - \hat{p} - \sqrt{2H\hat{r}} \tag{25}$$

where here V is the absolute value of the square root. Expression (25) is negative irrespective of the sign of $\hat{p}$ and stays so for $\hat{r} < 2H$ because $\hat{p}$ is no longer singular. As a consequence p_c below $2H$ acquires the imaginary part $i\pi\hat{r}$. Below $\hat{r} = 2M$ the denominator of the argument of the logarithm in eq.(16) becomes negative so that the argument of the logarithm reverts to positive values. Thus the so called classically forbidden region where p_c becomes complex is $2M < \hat{r} < 2H$ independent of m and of the deformation g and the integral of the imaginary part of p_c for any deformation g and for any mass m of the shell is

$$\mathrm{Im}\int p_c dr = \pi\int_{2M}^{2H} r dr = 2\pi(H^2 - M^2) = \frac{\Delta S}{2} \tag{26}$$

which is the original result derived in [1, 11] in the zero mass case. Parikh and Wilczek [11] gave to

$$\exp(-2\mathrm{Im}\int p_c dr) \tag{27}$$

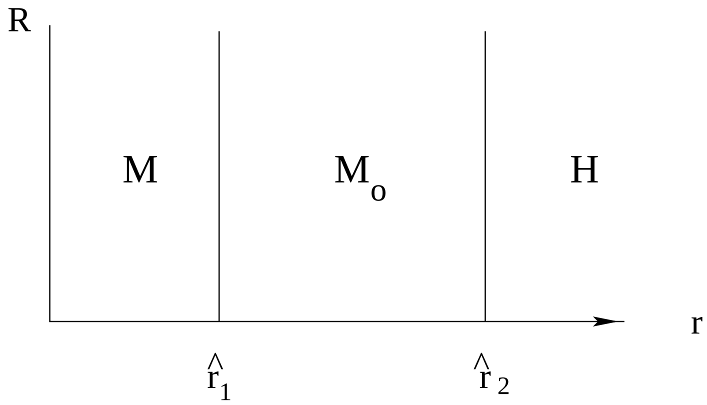

Figure 4. Two shell dynamics

the interpretation of the tunneling probability for the emission of a quantum of energy ω. Criticism and alternative proposals for the emission probability like

$$\exp(-\mathrm{Im}\oint p_c dr) \tag{28}$$

$$\exp\left(-2\mathrm{Im}(\int p_c dr + \text{temporal contribution})\right) \tag{29}$$

followed [12, 13, 14, 15, 16, 17, 18, 19, 20, 29, 30, 31]. Here instead we shall discuss the role of eq.(16) in the framework of mode analysis. This will be done in Sec.(8).

5. Two shell reduced action

Now we have three characteristic masses, M, H and an intermediate mass M_0, see Fig.4, which can change only if the two shells cross. Working as before with $M = \text{const}$ considered as a datum of the problem we reach the reduced action [3]

$$\begin{aligned}&\dot{\hat{r}}_1 p_{c1} + \dot{\hat{r}}_2 p_{c2} + \dot{H}(R(\hat{r}_1) - \hat{r}_1)\frac{\partial T}{\partial H}\mathcal{D} + \dot{M}_0(R(\hat{r}_1) - \hat{r}_1)\frac{\partial T}{\partial M_0}\mathcal{D} + \\ &+\frac{d}{dt}\int_{r_0}^{\hat{r}_2} F dr - \dot{M}_0 \int_{\hat{r}_1}^{\hat{r}_2} \frac{\partial F}{\partial M_0} dr + (-N^r \pi_L + NRR')|_{r_0}^{r_m}\end{aligned} \tag{30}$$

where

$$T = \log\frac{V_2}{R(\hat{r}_2)}; \qquad \mathcal{D} = R(\Delta\mathcal{L} - \Delta\mathcal{B})|_{\hat{r}_1}; \tag{31}$$

$$p_{c1} = R'(\hat{r}_1 + \varepsilon)\mathcal{D}; \qquad p_{c2} = p^0_{c2} + \frac{d}{d\hat{r}_2}(R(\hat{r}_1) - \hat{r}_1)\mathcal{D} \tag{32}$$

and p^0_{c2} is given by eq.(16) with M replaced by M_0. The novelty is that now even in the outer gauge the time derivative of H intervenes in addition to $\dot{M}_0$ and p_{c1} and p_{c2} depend both on $\hat{r}_1$ and $\hat{r}_2$. We can vary $\hat{r}_1$, $\hat{r}_2$, H and M_0 independently obtaining the correct equations of motion [3]. As expected one finds that the exterior shell moves irrespective of the dynamics which develops at lower values of r until a crossing occurs.

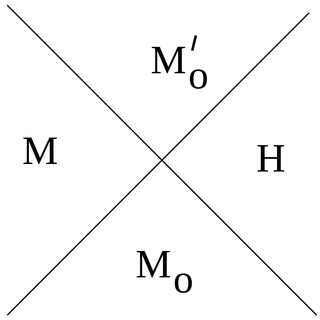

Figure 5. The exchange diagram

6. The exchange relations

In case of crossing of the two shells from the equations of Sec.(5) we can obtain relations between $M, M_0, H, \hat{r}_e$ and M'_0 being $\hat{r}_e$ the the shell position at the crossing and M'_0 the intermediate mass after the crossing. During the crossing the masses of the shells can change, provided they satisfy a relation analogous to the energy-momentum conservation is special relativity

$$\hat{p}_1 + \hat{p}_2 = \hat{p}'_1 + \hat{p}'_2 \tag{33}$$
$$V_1 + V_2 = V'_1 + V'_2; \quad V_n = \sqrt{\hat{p}_n^2 + m_n^2}; \quad V'_n = \sqrt{\hat{p'}_n^2 + m'^2_n}.$$

This is an outcome of the constraints. It is not possible to predict the final masses of the two shells as it depends on the details of the interaction which has to be specified. A relatively simple case is when the masses are unchanged during the crossing (transparent crossing) [3] and the simplest case is the crossing of two massless shells which remain massless, thus re-obtaining the well known Dray-'t Hooft-Redmount relations [21, 22] which we report here below

$$H\hat{r}_e + M\hat{r}_e - 2HM = M_0\hat{r}_e - 2M_0M'_0 + M'_0\hat{r}_e \tag{34}$$

being $\hat{r}_e$ the crossing radius.

7. Integrability of the form $p_{c1}d\hat{r}_1 + p_{c2}d\hat{r}_2$ and imaginary part of the space-component of the action

The space-part of the on-shell action for two massive shells of matter is given from eq.(30) by

$$\int_{t_i}^{t_f} (p_{c1}\ \dot{\hat{r}}_1 + p_{c2}\ \dot{\hat{r}}_2)\ dt \tag{35}$$

as on the equations of motion $\dot{H}(t) = \dot{M}(t) = 0$. It is possible to prove a theorem analogous to the one found in the books of Whittaker and Arnold [23, 24] i.e. that in presence of a constant of motion, in addition to the hamiltonian, the form $p_{c1}d\hat{r}_1 + p_{c2}d\hat{r}_2$ is closed even though the proof is somewhat different [3]. The intermediate mass M_0 plays the role of the additional constant of motion. The above result allows to deform the integration path as to bring $\hat{r}_1$ immediately below or above $\hat{r}_2$. This is allowed by the absence of discontinuities in our scheme. But, in words, two coaleshed shell have the same properties of a single shell with energy $H - M$. The final result is that

$$\mathrm{Im}\int_{t_i}^{t_f} dt(p_{c1}\ \dot{\hat{r}}_1 + p_{c2}\ \dot{\hat{r}}_{c2}) = \mathrm{Im}\int_{r_{1i},r_{2i}}^{r_{1f},r_{2f}} (p_{c1}\ d\hat{r}_1 + p_{c2}\ d\hat{r}_2) = 2\pi(H^2 - M^2). \tag{36}$$

The result (36) holds also for in the case when at the crossing the two shells can change their mass. For details we refer to [3].

8. Mode analysis

The original way to extract information on the spectrum of the radiation is mode analysis [26, 1, 27]. In the massless case

$$\int^{\hat{r}} p_c d\hat{r}' = f(\hat{r}, M) - f(\hat{r}, H) \tag{37}$$

can be computed exactly and its expansion to first order in $\omega = H - M$ is

$$\int^{\hat{r}} p_c d\hat{r}' = 4M\omega \log(\hat{r} - 2M) + \text{ regular terms.} \tag{38}$$

Given the semiclassical mode

$$\phi(r, t) = e^{i\int^{\hat{r}} p_c d\hat{r}' - i\omega t} \tag{39}$$

we can perform the analysis of the mode regular at the horizon in terms of the above mode. This analysis can be performed either by scalar product [25] i.e. space integration where one has to keep into account that the background metric is Painlevé or by time Fourier analysis. In this way one obtains the well known formulas for the Bogoliubov coefficients. The regular outgoing modes near the horizon are given by

$$\psi(\hat{r}, t) = e^{ik(\hat{r}-2M)e^{-\frac{t}{4M}}}. \tag{40}$$

Computing the scalar product, and taking into account that the background metric is the Painlevé metric, we have

$$-i \int (\psi^* \partial_\rho \phi - \phi \partial_\rho \psi^*) g^{\rho 0} \varepsilon_{0r\theta\phi} \sqrt{-g}\; d\hat{r} d\theta d\phi \tag{41}$$

with

$$g^{rt} = N^r = \sqrt{\frac{2M}{\hat{r}}} \quad \text{and} \quad \sqrt{-g} = 1 \tag{42}$$

and the integration region is outside the horizon. Keeping only the most singular terms at the horizon eq.(41) reduces, with $\tau = \exp(-t/4M)$, to

$$\int_0^\infty e^{-ikx\tau} e^{4iM\omega \log x - i\omega t}\; \frac{dx}{x}. \tag{43}$$

We can compute such scalar products at $t = 0$ obtaining the standard Hawking integral

$$\int_0^\infty e^{-ikx} e^{4iM\omega \log x}\; \frac{dx}{x} = e^{2\pi\omega M} (k)^{-4i\omega M} \Gamma(4i\omega M) = \text{const } \alpha^*_{k\omega} \tag{44}$$

which gives the dominant contribution for large k. The coefficient $\beta_{k\omega}$ is obtained by changing in (44) ω into $-\omega$. Alternatively we can extract the Bogoliubov coefficients by performing a time Fourier transform i.e.

$$\int_{-\infty}^{+\infty} e^{-ikx\tau} e^{4iM\omega \log x - i\omega t} dt = \int_0^{+\infty} e^{-ikx\tau} e^{4iM\omega \log(x\tau)}\; \frac{d\tau}{\tau} \tag{45}$$

which is, as it should be, a result independent of r (i.e. x) and reproduces eq.(44). The derivation is valid to first order in ω which is the realm of validity of the external field approximation. For

finite ω it is problematic to perform a space integration on modes, because we have not a well defined background metric.
Thus Kraus and Wilczek [1] followed the time Fourier analysis method. In order to do so one has to construct the non-perturbative modes regular at the horizon. This is not a completely trivial task. The regular modes are given by

$$e^{iS} = e^{ik\hat{r}(0)+i\int_{\hat{r}(0)}^{r} p_c d\hat{r} - i(H-M)t} \tag{46}$$

where S is the on shell action computed with the following boundary conditions: 1) At time t the shell position is $\hat{r}$, outside the horizon. 2) At time $t=0$ the conjugate momentum is k, i.e. $S(0,\hat{r}) = k\hat{r}$. Due to these boundary conditions $\hat{r}(0)$, H and as a consequence p_c depend on $k, t, \hat{r}$ even if along each trajectory H is always a constant of motion. Using the saddle point approximation [1, 27] one obtains that the absolute value of the Bogoliubov coefficient $\alpha_{k\omega}$ is given by

$$\left| e^{i\int_{r(0)}^{r} p_c(r',H,M)dr'} \right| = e^{-\mathrm{Im}\int_{r(0)}^{r} p_c(r',H,M)dr'} \tag{47}$$

computed for $H = M + \omega$. What is important here is that only the space part of the action appears, as the time part $(H-M)t$ cancels with ωt at the saddle point. $\hat{r}(0)$ is given by the condition

$$k = \hat{r}(0)\log\frac{\sqrt{\hat{r}(0)} - \sqrt{2M}}{\sqrt{\hat{r}(0)} - \sqrt{2H}} \tag{48}$$

which for $H = M + \omega > M$ is solved by

$$2M < 2H < \hat{r}(0) \tag{49}$$

and thus there is no imaginary contribution to the integral from the gap $2M, 2H$ as we discussed in Sect.(4). The Bogoliubov coefficient $\beta_{k\omega}$ is obtained by changing in (47) ω in $-\omega$ (always $\omega > 0$). Then we have

$$\hat{r}(0) < 2H < 2M \tag{50}$$

The value of the saddle point t, now complex, is given by

$$t = 4H\log\frac{\sqrt{\hat{r}} - \sqrt{2H}}{\sqrt{\hat{r}(0)} - \sqrt{2H}} \tag{51}$$

and the r.h.s. term of eq.(47) becomes

$$e^{-4\pi M\omega(1-\omega/2M)} \tag{52}$$

where the last passage is due to eq.(26) for which we gave a general proof within the family of Painlevé gauges even in the massive case. An explicit derivation of the result (52) has been given in [10] by working out the late time expansion of the time development of the action. The main point is that in the mode analysis what comes in is not the total action of a model particle crossing the horizon, but only the “space part” of it. The analysis and the results depend on the interpretation of the expression

$$\exp(iS) \tag{53}$$

as modes of the field dressed by the gravitational interaction with non vanishing quanta of energy. The result eq.(36) proven for the emission of two shells [3] was interpreted [28] as the absence of correlations among quanta in the emitted radiation. It would be of interest to give a similar “mode interpretation” of the result (36) derived for the emission of two shells which during the time evolution can also interact [3]. To this end one should compute the two shell modes which are regular at the horizon, and perform a time Fourier analysis of them. This has not yet been accomplished.

9. Concluding remarks

In this paper we gave a general treatment of the dynamics of one or more self- gravitating spherical shell of matter in a spherical gravitational field. We extended the treatment of [1] to more than one shell [3] proving on the way the universality of the reduced conjugate momentum within the family of Painlevé gauges [10].

This allows to give a general derivation of the exchange relations and also, exploiting an integrability result, to extend the result on the imaginary part of the space part of the action to more than one shell. Instead of following the tunneling picture here we have revisited the treatment which follows by interpreting the exponential of the action, properly subtracted, as the dressed modes of the Hawking radiation. The main point here is that according such a mode interpretation only the space part of the action and in particular its imaginary part comes in determining the Bogoliubov coefficient and thus in determining the Hawking spectrum corrected for the back reaction effects.

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